MENIIT
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JEE MAIN-2021

COMPUTER BASED TEST (CBT)

DATE : 22-07-2021 (EVENING SHIFT) | TIME : (3.00 pm to 6.00 pm)

Duration 3 Hours | Max. Marks : 300

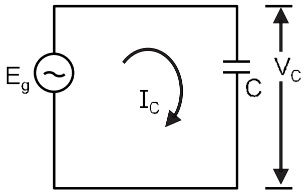
QUESTION & SOLUTIONS

PART A : PHYSICS

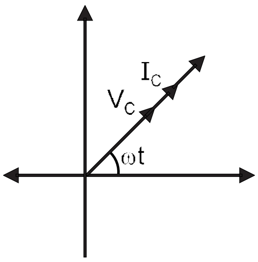
Single Choice Type

This section contains **20 Single choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. In a circuit consisting of a capacitance and a generator with alternating emf $E_g = E_{g0}$ V_C and I_C are the voltage and current. Correct phasor diagram for such circuit is :



(1)



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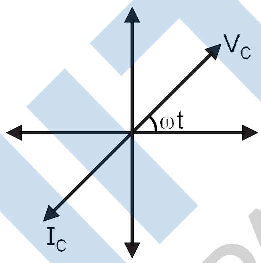

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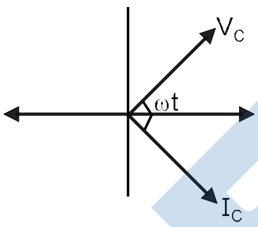

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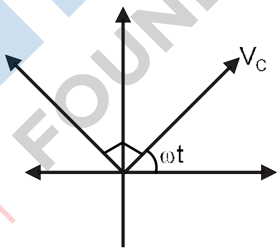


Ans. 4

Sol. In pure capacitive circuit, current leads voltage by $\pi/2$ phase.

2. Intensity of sunlight is observed as 0.092 Wm^{-2} at a point in free space. What will be the peak value of magnetic field at that point ? ($\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$).

(1) 1.96×10^{-8} T (2) 5.88 T (3) 2.77×10^{-8} T (4) 8.31 T

Ans. 3

Sol. $\frac{1}{2}\ {}_0E_0^2\ C\ \ I$

$E_0^2\ \ \frac{2I}{{}_0C}$ $\qquad$ $E_0\ \ \sqrt{\frac{2I}{{}_0C}}$

$B_0\ \ \frac{E_0}{C}\ \ \frac{1}{C}\sqrt{\frac{2I}{{}_0C}}\ \ 2.77\ \ 10^{\ 8}\ T$

3. Choose the correct option :

(1) True dip is always equal to apparent dip.

(2) True dip is less than the apparent dip.

(3) True dip is always greater than the apparent dip.

(4) True dip is not mathematically related to apparent dip.

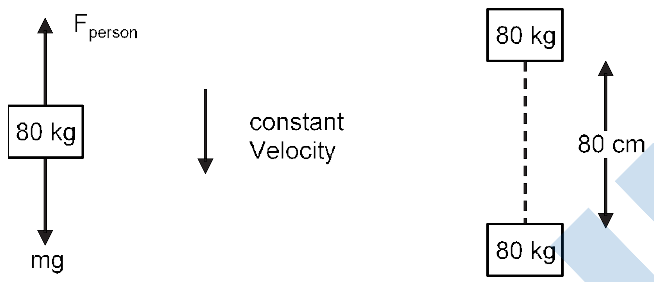
Ans. 2

4. A porter lifts a suitcase of mass 80 kg and at the destination lowers it down by a distance of 80 cm with a constant velocity. Calculate the work done by the porter in lowering the suitcase. (Take g = 9.8 ms^{-2})

(1) 784.0 J (2) – 62700.0 J (3) 627.2 J (4) – 627.2 J

Ans. 4

Sol.



$F_{person} = mg = 784$ N

$W_{person} = F_p s \cos 180° = -784 \times 80 \times 10^{-2} = -627.2$ J

5. T_0 is the time period of a simple pendulum at a place. If the length of the pendulum is reduced to $\frac{1}{16}$ times of its initial value, the modified time period is :

(1) T_0 (2) $\frac{1}{4}T_0$ (3) $8\pi T_0$ (4) $4T_0$

Ans. 2

Sol. $T \quad 2\sqrt{\frac{\ell}{g}}$

$T' \quad 2\sqrt{\frac{\ell}{16g}} \quad \frac{T}{4}$

6. Consider a situation in which a ring, a solid cylinder and a solid sphere roll down on the same inclined plane without slipping, Assume that they start rolling from rest and having identical diameter. The correct statement for this situation is :

(1) The sphere has the greatest and the ring has least velocity of the centre of mass at the bottom of the inclined plane.

(2) The ring has the greatest and the cylinder has the least velocity of the centre of mass at the bottom of the inclined plane.

(3) All of them will have same velocity

(4) The cylinder has the greatest and the sphere has the least velocity of the centre of mass at the bottom of the inclined plane.

Ans. 1

Sol. $mgh \quad \frac{1}{2} I_{cm} \quad ^2 \quad \frac{1}{2} mv^2$

$V \quad \sqrt{\dfrac{2gh}{1 \quad \dfrac{I_{cm}}{mR^2}}}$

As $I\uparrow$, $V\downarrow$

7. What will be the average value for a monoatomic gas in thermal equilibrium at temperature T?

(1) $\frac{1}{2}k_BT$ (2) $\frac{3}{2}k_BT$ (3) $\frac{2}{3}k_BT$ (4) k_BT

Ans. 2

8. Match List-I with List-II :

List-I	List-II
(a) $L \quad \frac{1}{C}$	(i) Current is in phase with emf
(b) $L \quad \frac{1}{C}$	(ii) Current lags behind the applied emf
(c) $L \quad \frac{1}{C}$	(iii) Maximum current occurs
(d) Resonant frequency	(iv) Current leads the emf

Choose the correct answer from the option given below:

(1) (a)-(iv); (b)-(iii); (c)-(ii); (d)-(i)

(2) (a)-(ii); (b)-(i); (c)-(iv); (d)-(iii)

(3) (a)-(ii); (b)-(i); (c)-(iii); (d)-(iv)

(4) (a)-(iii); (b)-(i); (c)-(iv); (d)-(ii)

Ans. 2

9. Consider a situation in which reverse biased current of a particular P-N junction increases when it is exposed to a light of wavelength $\leq$ 621 nm. During this process, enhancement in carrier concentration takes place due to generation of hole-electron pairs. The value of band gap is nearly.

(1) 4eV (2) 1 eV (3) 2 eV (4) 0.5 eV

Ans. 3

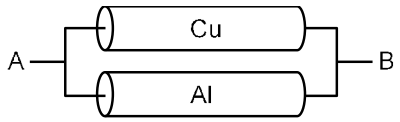
Sol. $E \quad \frac{hc}{} \quad eV$

$V \quad \frac{hc}{e}$

$\frac{1224\,ev \quad nm}{e \quad 612nm} \quad 2\,volt$

Band gap = 2eV

10. A copper (Cu) rod of length 25 cm and cross-sectional area 3mm2 is joined with a similar Aluminium (Al) rod as shown in figure. Find the resistance of the combination between the ends A and B. (Take Resistivity of Copper = 1.7×10^{-8} Ωm)



(1) 0.858 mΩ (2) 0.0858 mΩ (3) 2.170 mΩ (4) 1.420 mΩ

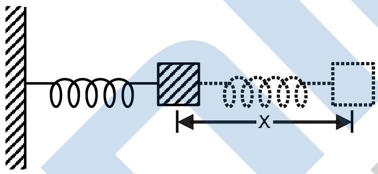
Ans. 1

Sol. $R_{eq}\ \ \frac{R_1R_2}{R_1\ \ R_2}\ \ \frac{{}_1\frac{\ell}{A}\ \ {}_2\frac{\ell}{A}}{{}_1\frac{\ell}{A}\ \ \ \ {}_2\frac{\ell}{A}}$

$\frac{\ell}{A}\ \frac{{}_1\ {}_2}{{}_1\ \ \ {}_2}\ \ \frac{0.25}{3\ \ 10^{\ 6}}\ \frac{1.7\ \ 10^{\ 6}\ \ 2.6\ \ 10^{\ 8}}{1.7\ \ 10^{\ 8}\ \ 2.6\ \ 10^{\ 8}}$

$= 0.085 \times 10^{-2} = 0.858$ mΩ

11. The motion of a mass on a spring, with spring constant K is as shown in figure.



The equation of motion is given by x(t) = Asinωt + Bcosωt $\sqrt{\frac{K}{m}}$

Supper that at time t = 0, the position of mass is x(0) and velocity υ(0), then its displacement can also be represented as x(t) = Ccos(ωt – ϕ), where C and ϕ are :

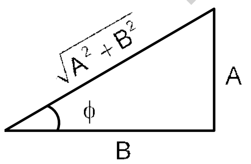
(1) C $\sqrt{\frac{(0)^2}{2}\ \ x(0)^2}$, tan $^{1}\ \frac{(0)}{x(0)}$ (2) C $\sqrt{\frac{2\ (0)^2}{2}\ \ x(0)^2}$, tan $^{1}\ \frac{x(0)}{2\ (0)}$

(3) C $\sqrt{\frac{2\ (0)^2}{2}\ \ x(0)^2}$, tan $^{1}\ \frac{(0)}{x(0)}$ (4) C $\sqrt{\frac{(0)^2}{2}\ \ x(0)^2}$, tan $^{1}\ \frac{x(0)}{(0)}$

Ans. 1

Sol. x(t) = Asinωt + Bcosωt v(t) = Aωcosωt – Bωsinωt

x(0) = B , v(0) = Aω



$\sqrt{A^2\ \ ^2}$ cos(t)

$C \quad \sqrt{A^2 \quad B^2} \qquad C \quad \sqrt{\frac{(0)^2}{2} \quad x(0)^2}$

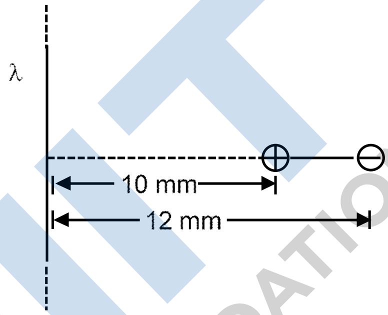
$\tan^{1}\frac{A}{B} \qquad \tan^{1}\frac{(0)}{x(0)}$

12. An electric dipole is placed on x-axis in proximity to a line charge of linear charge density 3.0×10^{-6} C/m. Line charge is placed on z-axis and positive and negative charge of dipole is at a distance of 10 mm and 12 mm from the origin respectively. If total force of 4 N is exerted on the dipole, find out the amount of positive or negative charge of the dipole.

(1) 0.485 mC (2) 4.44 μC (3) 815.1 nC (4) 8.8 μC

Ans. 2

Sol. Let charge be Q



Net force $\frac{2k \quad Q}{r_1} \quad \frac{2k \;(\;Q)}{r_2}$

$4N \quad 2k \quad Q \quad \frac{1}{r_1} \quad \frac{1}{r_2}$

$4N \quad 2 \quad 9 \quad 10^9 \quad 3 \quad 10^{6}Q \quad \frac{1}{10} \quad \frac{1}{12} \quad 10^3$

$4N \quad 54 \quad 10^6 Q \quad \frac{1}{60}$

$Q = 4.44 \times 10^{-6}$ C = 4.44 μC

13. What will be the projection of vector $\vec{A} \quad \hat{i} \quad \hat{j} \quad \hat{k}$ on vector $\vec{B} \quad \hat{i} \quad \hat{j}$?

(1) $2 \;\hat{i} \quad \hat{j} \quad \hat{k}$ (2) $\sqrt{2} \;\hat{i} \quad \hat{j}$ (3) $\hat{i} \quad \hat{j}$ (4) $\sqrt{2} \;\hat{i} \quad \hat{j} \quad \hat{k}$

Ans. 3

Sol. $|\vec{A}|\cos \quad \vec{B}$

$\frac{\vec{A} \quad \vec{B}}{|\vec{B}|}\hat{B} \quad \hat{i} \quad \hat{j}$

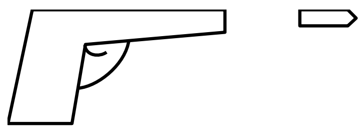
14. A bullet of '4 g' mass is fired from a gun of mass 4 kg. If the bullet moves the muzzle speed of 50 ms^{-1}, the impulse imparted to the gun and velocity of recoil of gun are :

(1) 0.4 kg ms^{-1}, 0.1 ms^{-1} (2) 0.2 kg ms^{-1}, 0.1 ms^{-1}

(3) 0.2 kg ms^{-1}, 0.05 ms^{-1} (4) 0.4 kg ms^{-1}, 0.05 ms^{-1}

Ans. 3

Sol. Initial momentum = final momentum



$0 = m_G V_G + M_B V_b$

$\Rightarrow \quad 0 \quad 4 \quad V_G \quad \frac{4}{1000}V_b$

$\Rightarrow \quad V_b = -1000\, V_G$ ....(1)

$V_{bG} = V_b - V_G$

$\Rightarrow \quad 50 = V_B - V_G$ ....(2)

$\Rightarrow \quad 50 = -1001\ V_G$

$V_G \approx 0.05$ m/s

impulse = $m_G V_G = 4 \times 0.05 = 0.2$ kg m/s

15. **Statement-I :** The ferromagnetic property depends on temperature. At high temperature, ferromagnet becomes paramagnet.

Statement-II : At high temperature, the domain wall area of a ferromagnetic substance increases.

In the light of the above statements, choose the most appropriate answer from the option given below:

(1) Both Statement-I and Statement-II are true (2) Statement-I is false but Statement-II is true

(3) Both Statement-I and Statement-II are false (4) Statement-I is true but Statement-II is false

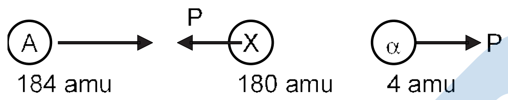
Ans. 4

16. A nucleus with mass number 184 initially at rest emits an □-particle. If the Q value of the reaction is 5.5 MeV, calculate the kinetic energy of the α-particle.

(1) 5.5 MeV (2) 5.0 MeV (3) 5.38 MeV (4) 0.12 MeV

Ans. 3

Sol.



$k \quad \frac{A \quad 4}{A} \quad Q \quad 5.38$ MeV

17. What should be the height of transmitting antenna and the population covered if the television telecast is to cover a radius of 150 km? The average population density around the tower is 2000/ km^2 and the value of Re = 6.5 × 106 m.

(1) Height = 1731 m, Population Covered = 1413 × 10^5

(2) Height = 1600 m, Population Covered = 2 × 10^5

(3) Height = 1800 m, Population Covered = 1413 ×10^8

(4) Height = 1241 m, Population Covered = 7 × 10^5

Ans. 1

Sol. Radius of earth = 6400 km

d = 150 km

height of Antena = ?

$d \quad \sqrt{2Rh}$

$h \quad \frac{d^2}{2R} \quad \frac{150 \quad 150 \quad 10^6}{2 \quad 6.5 \quad 10^6} \quad 1730.7 \quad 1731\,\text{m}$

Population covered $\Rightarrow 2\pi Rh \times$ density

$2\pi \times 6.5 \times 106 \times 1730.7 \times 2000 \times 10^{-6} \approx 1413 \times 10^5$

18. An electron of mass m_e and a proton of mass m_p are accelerated through the same potential difference. The ratio of the de-Broglie wavelength associated with the electron to that with the proton is:

(1) $\frac{m_p}{m_e}$ (2) 1 (3) $\frac{m_e}{m_p}$ (4) $\sqrt{\frac{m_p}{m_e}}$

Ans. 4

Sol. $\frac{h}{p} \quad \frac{h}{\sqrt{2mqV}}$

$$\frac{{}_e}{{}_p} \quad \sqrt{\frac{m_p}{m_e}}$$

19. A body is projected vertically upwards from the surface of earth with a velocity sufficient enough to carry to infinity. The time taken by it to reach height h is______s.

(1) $\sqrt{\frac{2R_e}{g}} \quad 1 \quad \frac{h}{R_e}^{\frac{3}{2}} \quad 1$

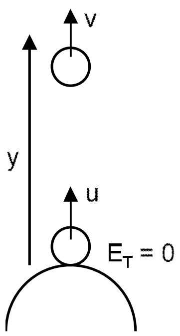
(2) $\sqrt{\frac{R_e}{2g}} \quad 1 \quad \frac{h}{R_e}^{\frac{3}{2}} \quad 1$

(3) $\frac{1}{3}\sqrt{\frac{R_e}{2g}} \quad 1 \quad \frac{h}{R_e}^{\frac{3}{2}} \quad 1$

(4) $\frac{1}{3}\sqrt{\frac{2R_e}{g}} \quad 1 \quad \frac{h}{R_e}^{\frac{3}{2}} \quad 1$

Ans. 4

Sol. $\frac{1}{2}mv^2 \quad \frac{GMm}{R \quad y} \quad 0$



$$v \quad \sqrt{\frac{2GM}{R \quad y}}$$

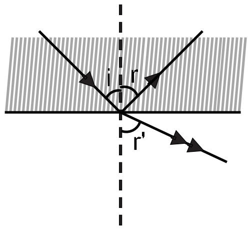
$$\frac{dy}{dt} \quad \sqrt{\frac{2GM}{R \quad y}}$$

$$\int_0^y \sqrt{R \quad y}dy \quad \sqrt{2Gm}\int_0^t dt \qquad \frac{2(R \quad y)^{\frac{3}{2}}}{3}\Bigg|_0^y \quad \sqrt{2Gm}\ t$$

$$\frac{2}{3} \quad (R \quad y)^{\frac{3}{2}} \quad R^{\frac{3}{2}} \quad \sqrt{2Gm}\ t \qquad t \quad \frac{\frac{2}{3} \quad (R \quad y)^{\frac{3}{2}} \quad R^{\frac{3}{2}}}{\sqrt{2Gm}}$$

$$\frac{2}{3}\sqrt{\frac{2RLe}{g}} \quad 1 \quad \frac{h}{R_e}^{\frac{3}{2}} \quad 1$$

20. A ray of light passes from a denser medium to a rarer medium at an angle of incidence i. The reflected and refracted rays make an angle of 90° with each other. The angle of reflection and refraction are respectively r and r'. The critical angle is given by:



(1) $\sin^{-1}(\tan r)$ (2) $\sin^{-1}(\cot r)$ (3) $\tan^{-1}(\sin i)$ (4) $\sin^{-1}(\tan r')$

Ans. 1

Sol. $i = r \text{ \& } r' = 90 - r$

$$_{C} \quad \sin^{1}\frac{1}{\ }$$

$$\sin^{1}\frac{\sin i}{\sin r'} \quad \sin^{1}\frac{\sin i}{\sin r} \quad \sin^{1}(\tan r)$$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. The area of cross-section of a railway track is 0.01 m^2. The temperature variation is 10°C. Coefficient of linear expansion of material of track 10–5/°C. The energy stored per meter in the track is____J/m. (Young's modulus of material of track is 10^{11} Nm^{-2})

Ans. 5

Sol. $U \ \frac{1}{2} \ \text{stress} \ \ \text{strain} \ \ \text{volume}$

$U \ \frac{1}{2} \ \text{stress} \ \ \text{strain} \ \ \text{volume} \ \ A\ell$

$\frac{U}{\ell} \ \frac{1}{2} \ \text{stress} \ \ \text{strain} \ \ \text{volume} \ \ A$

$\frac{1}{2} \ Y \ \ (\text{strain})^2 \ \ A$

$\frac{1}{2} \ Y \ \ \frac{\ell}{\ell}^{\ 2} \ \ A \ \ \frac{1}{2} Y \ \frac{\ell \ \ t}{\ell}^{\ 2} \ \ A \ \ \frac{1}{2} Y \ ^2 \ t^2 A$

$\frac{1}{2} \ 10^{11} \ \ 10^{\ 10} \ \ 10 \ \ 10 \ \ 10^{\ 2} \ \ 5 \text{ Jule / m}$

2. In 5 minutes, a body cools from 75°C to 65°C at room temperature of 25°C. The temperature of body at the end of next 5 minutes is _____°C.

Ans. 57

Sol. $\frac{T}{t} \ \ K \ \frac{T_1 \ \ T_2}{2} \ \ T_0$

$\frac{75 \ \ 65}{5} \ \ K \ \frac{75 \ \ 65}{2} \ \ 25$ …(1)

$\frac{65 \ \ T}{5} \ \ K \ \frac{T \ \ 65}{2} \ \ 25$ …(2)

Eq(2)/Eq(1)

$\frac{65 \ \ T}{75 \ \ 65} \ \ \frac{\frac{T \ \ 65}{2} \ \ 25}{\frac{75 \ \ 65}{2} \ \ 25}$

$\frac{65 \ \ T}{10} \ \ \frac{T \ \ 15}{90}$

90 × 65 – 90 T = 10 T + 10 × 15

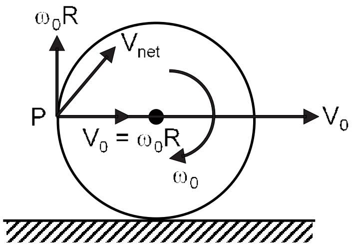
100 T = 90 × 65 – 15 × 10

T = 57°C

3. The centre of a wheel rolling on a plane surface moves with a speed υ_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed $\sqrt{x}\,\upsilon_0$. Then the value of x is____.

Ans. 2

Sol.



$V_P = V_{net} = \sqrt{V_0^2 + (\omega_0 R)^2} = \sqrt{V_0^2 + V_0^2} = \sqrt{2}V_0$

4. The position of the centre of mass of a uniform semi-circular wire of radius 'R' placed in x-y plane with its centre at the origin and the line joining its ends as x-axis is given by $\left(0, \frac{xR}{\pi}\right)$. Then, the value of x is___.

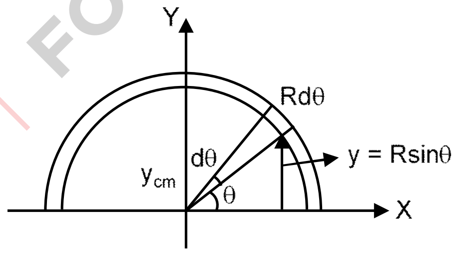
Ans. 2

Sol. To find ycm we use $y_{cm} = \frac{1}{M}\int dm\,y$ .....(i)

Here for dm we consider an elemental arc of the ring at an angle θ from the x-direction of angular width dθ. If radius of the ring is R then its y coordinate will be R sinθ, here dm is given as

$$dm = \frac{M}{\pi R} R\,d\theta$$

So from equation ....(i), we have



$$y_{cm} = \frac{1}{M}\int_0^{\pi} \frac{M}{\pi R} R\,d\theta\,(R\sin\theta) = \frac{R}{\pi}\int_0^{\pi} \sin\theta\,d\theta$$

$$y_{cm} = \frac{2R}{\pi} \quad ....(ii)$$

$\therefore$ x = 2

5. Three particles P, Q and R moving along the vectors $\vec{A} = \hat{i} + \hat{j}$, $\vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is_____.

Ans. 3

Sol. $\vec{P} = K\left(\vec{A} \times \vec{B}\right)$

$$\begin{vmatrix} i & j & k \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{vmatrix}$$

$K \; \hat{i} \; \hat{j} \; \hat{k}$

$\vec{Q} \; I \vec{A} \; \vec{C}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 1 & 0 \end{vmatrix} \; 2\hat{k} \; I$$

$$\cos \; \frac{\vec{P} \; \vec{Q}}{|\vec{Q}||\vec{Q}|} \; \frac{1}{\sqrt{3}}$$

$$\cos^{\;1} \frac{1}{\sqrt{3}}$$

6. The total charge enclosed in an incremental volume of 2×10^{-9} m^3 located at the origin is ____nC, if electric flux density of its field is found as $D \;\; e^{\;x} \sin y \hat{i} \;\; e^{\;x} \cos y \hat{j} \;\; 2z\hat{k} \; C / m^2$.

Ans. $4 \varepsilon_0$

Sol. $\vec{V} \; \vec{E} \; \frac{}{_0}$

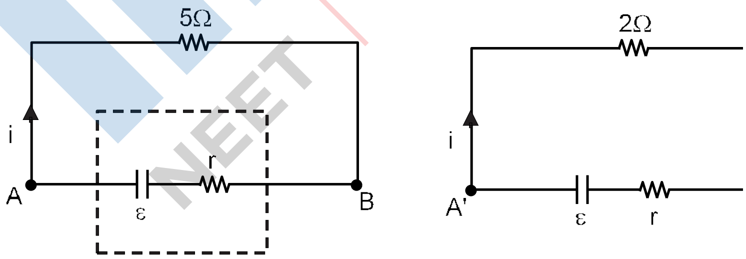
$$\frac{E}{x} \; \frac{E}{y} \; \frac{E}{z} \; \frac{}{_0}$$

$\rho = 2\varepsilon_0 \Rightarrow Q = 4\varepsilon_0$

7. In and electric circuit, a cell of certain emf provides a potential difference of 1.25 V across a load resistance of 5 Ω. However, it provides a potential difference of 1V across a load resistance of 2 Ω. The emf of the cell is given by $\frac{x}{10}$ V. Then the value of x is_____.

Ans. 15

Sol.



(1) (2)

(1) $I \; \frac{1.25}{5} \; 0.25 \; A$

$V_{AB} = 5V = \varepsilon - ir$

$1.25 = \varepsilon - 0.25\ r$ ...(1)

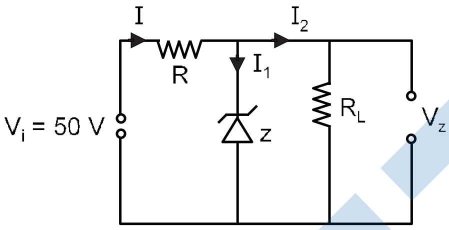
(2) $I \quad \frac{1}{2} \quad 0.5\ A$

$V_{A'B'} = 1 = \varepsilon - 0.5r$ ...(2)

Solving (1) & (2)

$\varepsilon = 15\ v$

8. In a given circuit diagram, a 5 V zener diode along with a series resistance is connected across a 50 V power supply. The minimum value of the resistance required, if the maximum zener current is 90 mA will be_____Ω.



Ans. 500

Sol. $\frac{45}{R} \quad 90\,mA$

$R \quad \frac{45}{90} \quad 10^3$

$R \leq 500\ \Omega$

9. Three students S_1, S_2 and S_3 perform an experiment for determining the acceleration due to gravity (g) using a simple pendulum. They use different lengths of pendulum and record time for different number of oscillations. The observations are shown in the table.

Student No.	Length of Pendulum (cm)	No. of oscillations (n)	Total time for n oscillations	Time Period (s)
1	64.0	8	128.0	16.0
2	64.0	4	64.0	16.0
3	20.0	4	36.0	9.0

(Least count of length = 0.1 cm

least count for time = 0.1 s)

If E_1, E_2 and E_3 are the percentage errors in 'g' for students 1, 2 and 3 respectively, then the minimum percentage error is obtained by student no.______.

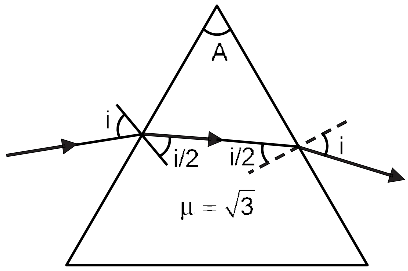
Ans. 1

Sol. Percentage error in 'g' $\frac{\ell}{\ell} \quad \frac{2\ T}{T} \quad 100$

10. A ray of light passing through a prism ($\sqrt{3}$) suffers minimum deviation. It is found that the angle of incidence is double the angle of refraction within the prism. Then, the angle of prism is_____(in degree).

Ans. 60

Sol.



$A \quad \frac{i}{2} \quad \frac{i}{2} \quad i$

$1\sin i \quad \sin\frac{i}{2}$

$2\sin\frac{i}{2}\cos\frac{i}{2} \quad \sqrt{3}\sin\frac{i}{2}$

$\cos\frac{i}{2} \quad \frac{\sqrt{3}}{2}$

$\frac{i}{2} \quad 30$

$i = 60°$

$\therefore \quad A = i = 60°$

PART B : CHEMISTRY

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. Which one of the following molecules does not show stereo isomerism?

(1) 3,4-Dimethylhex-3-ene (2) 3-Ethylhex-3-ene

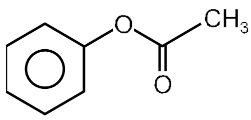
(3) 3-Methylhex-1-ene (4) 4-Methylhex-1-ene

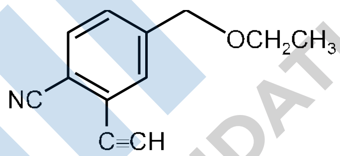
Ans. 2

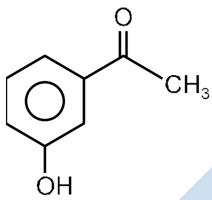
Sol. $CH_3-CH_2-\underset{\underset{CH_2-CH_3}{|}}{C}=C-CH_2-CH_3$

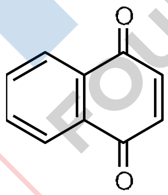
Neither show geometrical nor show optical isomerism.

2. Which one of the following compounds will provide a tertiary alcohol on reaction with excess of CH_3MgBr followed by hydrolysis?

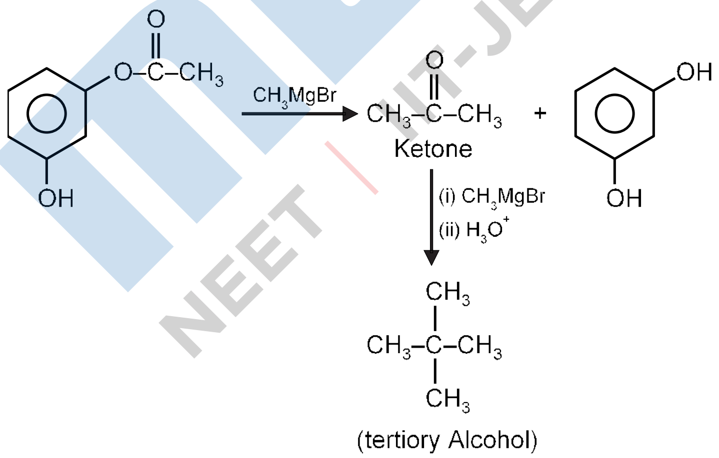
(1) 

(2) 

(3) 

(4) 

Ans. 1

Sol. 

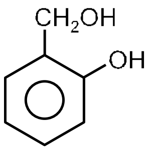
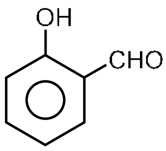
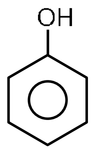
3. When $AgNO_3$ solution is added to KI solution then the sol produced is

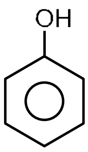
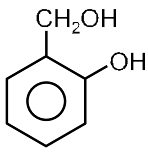
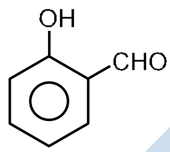
(1) AgI /Ag^+ (2) KI/NO_3^- (3) $AgNO_3/NO_3^-$ (4) AgI/I^-

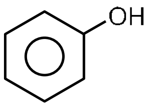
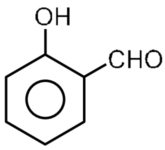
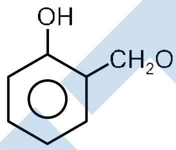
Ans. (4)

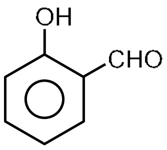
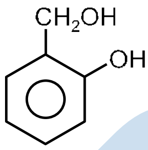
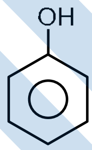
Sol. $AgNO_3 + KI \longrightarrow AgI/I^-$

4. An organic compound $A(C_6H_6O)$ gives dark green colouration with ferric chloride. On treatment with $CHCl_3$. and KOH followed by acidification gives compound B. Compound B can also be obtained from compound C on reaction with pyridinium chlorochromate (PCC).

(1) A =  B =  C = 

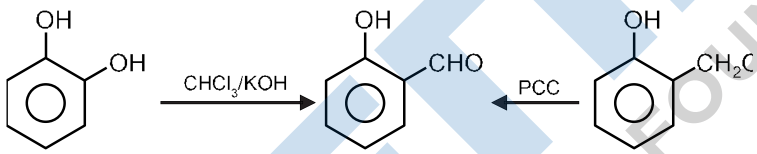
(2) A =  B =  C = 

(3) A =  B =  C = 

(4) A =  B =  C = 

Ans. 3

Sol.



(A) (B) (C)

5. Thiamine & pyridoxine are also known respectively as:

(1) Vitamin B_2 and Vitamin E (2) Vitamin B_1 and Vitamin B_6

(3) Vitamin B_6 and Vitamin B_2 (4) Vitamin E & Vitamin B_2

Ans. 2

Sol. NCERT

6. The set having ions which are coloured and paramagnetic both is:

(1) Cu^+, Zn^{2+}, Mn^{4+} (2) Sc^{3+}, V^{5+}, Ti^{4+} (3) Ni^{2+}, Mn^{7+}, Hg^{2+} (4) Cu^{2+}, Cr^{3+}, Sc^+

Ans. 4

Sol.

Ion	No. of unpaired e^-
Cu^{2+}	1
Sc^+	2
Cr^{3+}	3

This set is "paramagnetic & coloured"

7. Given below are the statements about diborane

(a) Diborane is prepared by the oxidation of $NaBH_4$ with I_2.

(b) Each boron atom is in sp^2 hybridized state.

(c) Diborane has one bridged 3 center-2 electron bond.

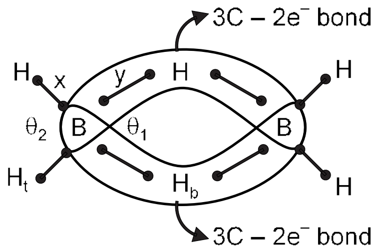
(d) Diborane is a planer molecule.

The option with correct statement(s) is:

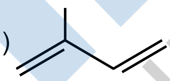
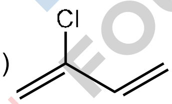
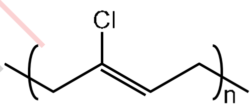
(1) (a) and (b) only (2) (a) only (3) (c) and (d) only (4) (c) only

Ans. 2

Sol. $2NaBH_4 \quad I_2 \quad ^{ether} \quad B_2H_6 \quad 2NaI \quad H_2$



8. Match List-I with List-II :

List-I	List-II
(a) Chloroprene	(i) 
(b) Neoprene	(ii) 
(c) Acrylonitrile	(iii) 
(d) Isoprene	(iv) CH_2=CH–CN

Choose the correct answer from the options given below:

(1) (a) - (ii), (b) - (i), (c) - (iv), (d) - (iii)

(2) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

(3) (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)

(4) (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Ans. 2

Sol. NCERT

9. Match List-I with List-II :

List-I	List-II
(a) Ba	(i) Organic solvent soluble compounds
(b) Ca	(ii) Outer electronic configuration $6s^2$
(c) Li	(iii) Oxalate insoluble in water
(d) Na	(iv) Formation of very strong monoacidic base

Choose the correct answer from the options given below:

(1) (a)–(iii), (b)–(ii), (c)-(iv) and (d)–(i) (2) (a)–(ii), (b)–(iii), (c)-(i) and (d)–(iv)

(3) (a)–(i), (b)–(iv), (c)-(ii) and (d)–(iii) (4) (a)–(iv), (b)–(i), (c)-(ii) and (d)–(iii)

Ans. 2

Sol. Ba - [Xe] $6s^2$

Ca - CaC_2O_4 is highly insoluble in water.

Li - Organic solvent soluble compounds due to covalent nature.

Na - Formation of very strong monoacidic base. eg. NaOH

10. Isotopes of hydrogen which emits low energy β– particle with half life greater than 12 year is/are :

(1) Tritium (2) Deuterium

(3) Deuterium and Tritium (4) Protium

Ans. 1

Sol. Only tritium is radioactive and emits low energy β particles ($t_{½}$, 12.33 years)

11. The water having more dissolved O_2 is:

(1) water at 4°C (2) polluted water (3) boiling water (4) water at 80°C

Ans. 1

Sol. Solubility of oxygen is increase with decrease in temperature.

12. Which purification technique is used for high boiling organic liquid compound (decomposes near its boiling point)?

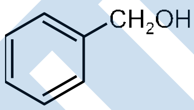
(1) Steam distillation (2) Fractional distillation

(3) Simple distillation (4) Reduced pressure distillation

Ans. 4

13. Which one of the following compounds does not exhibit resonance?

(1) $CH_3CH_2CH_2CONH_2$ (2) $CH_3CH_2OCH=CH_2$

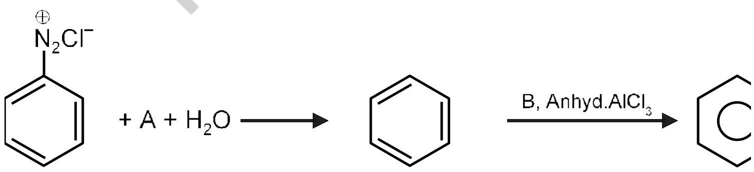
(3) 

(4) $CH_3CH_2CH=CHCH_2NH_2$

Ans. (4)

Sol. 

has no conjugation between π-bond and lone-pair hence there will be no resonance in this compound.

14. 

Major Product

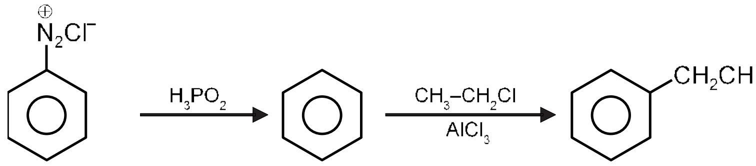
In the chemical reaction given above A & B respectively are:

(1) H_3PO_2 and CH_3CH_2Cl (2) CH_3CH_2Cl and H_3PO_2

(3) H_3PO_2 and CH_3CH_2OH (4) CH_3CH_2OH and H_3PO_2

Ans. 1

Sol.



15. Which one of the following statements for D.I. Mendeleeff, is incorrect?

(1) He authored the textbook-Principles of Chemistry.

(2) Element with atomic number 101 is named after him.

(3) At the time, he proposed Periodic Table of elements structure of atom was known.

(4) He invented accurate barometer.

Ans. 3

Sol. NCERT based.

Preliminary work for his great textbook "Principles of Chemistry" led Mendeleev to propose the Periodic Law and to construct his Periodic Table of elements. At that time, the structure of atom was unknown and Mendeleev's idea to consider that the properties of the elements were in someway related to their atomic masses was a very imaginative one.

You will notice from the modern Period Table (Fig.) that Mendeleev's name has been immortalized by naming the element with atomic number 101, as Mendelevium.

16. Match List-I with List-II:

List-I (Species)	**List-II (Hybrid orbitals)**
(a) SF_4	(i) sp^3d^2
(b) IF_5	(ii) d^2sp^3
(c) NO_2	(iii) sp^3d
(d) NH_4	(iv) sp^3
	(v) sp

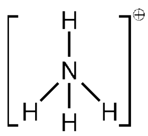
Choose the correct answer from the options given below:

(1) (a)–(ii), (b)–(i), (c)-(iv) and (d)–(v) (2) (a)–(iv), (b)–(iii), (c)-(ii) and (d)–(v)

(3) (a)–(i), (b)–(ii), (c)-(v) and (d)–(iii) (4) (a)–(iii), (b)–(i), (c)-(v) and (d)–(iv)

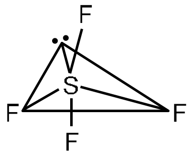
Ans. 4

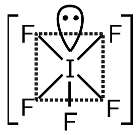
Sol. NH_4



sp^3

NO_2 O N O sp

SF_4  sp^3d

IF_5  sp^3d^2

17. Which one of the following group-15 hydride is the strongest reducing agent?

(1) BiH_3 (2) SbH_3 (3) AsH_3 (4) PH_3

Ans. 1

Sol. NH_3

PH_3

AsH_3

SbH_3

BiH_3

As we move down the group reducing power is increase.

18. Sulphide ion is soft base and its ores are common for metals.

(a) Pb (b) Al (c) Ag (d) Mg

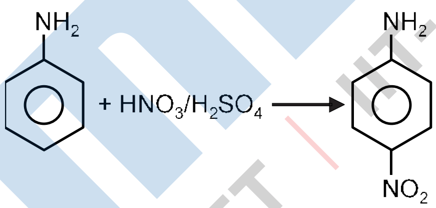
Choose the correct answer from the options given below:

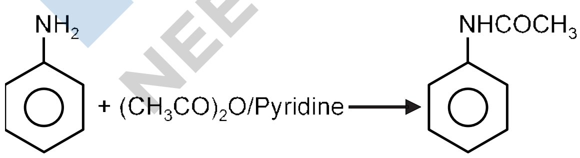
(1) (c) and (d) only (2) (a) and (d) only (3) (a) and (c) only (4) (a) and (b) only

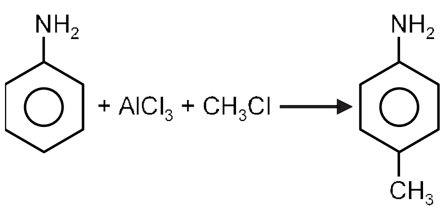
Ans. 3

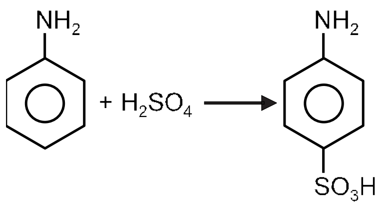
Sol. Sulphide ore ⇒ Pbs, Ag_2S, $CuFeS_2$, ZnS.

19. Which one of the following reactions does not occur?

(1) 

(2) 

(3) 

(4) 

Ans. 3

Sol. Aniline does not give Friedel craft reaction.

20. Which one of the following 0.06 M aqueous solutions has lowest freezing point?

(1) $C_6H_{12}O_6$ (2) $Al_2(SO_4)_3$ (3) KI (4) K_2SO_4

Ans. 2

Sol. $\Delta T_f = i\ K_f m$

Greater the i value lower will be freezing point

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Value of K_P for the equilibrium reaction $N_2O_4(g) \rightleftharpoons 2NO_2(g)$ at 288 K is 47·9. The K_C for this reaction at same temperature is __________. (Nearest integer)

($R = 0.083$ L bar K^{-1} mol^{-1})

Ans. 2

Sol. $K_P \quad K_C(RT)^{n_g}$

$47{\cdot}9 = KC\ (0{\cdot}083 \times 288)^1$

$K_C = 2$

2. Methylation of 10 g of benzene gave 9.2 g of toluene. Calculate the percentage yield of toluene __________. (Nearest integer)

Ans. 78

Sol. $\underset{\frac{10}{78}\text{ mole}}{C_6H_6} \xrightarrow{\text{Methylation}} \underset{\frac{10}{78}\text{ mole}}{C_6H_5CH_3}$

$(W_{theoritical}) \quad \frac{10}{78} \quad 92$

$\%\ \text{yield} \quad \frac{W_{actual}}{W_{theoritacal}} \quad 100$

$\frac{9.2}{10 \quad 92} \quad 78 \quad 100 \quad 78\%$

3. Assume a cell with the following reaction

$Cu(s) + 2Ag^{+}\ (1 \times 10^{-3}M) \rightarrow Cu^{2+}\ (0.250\ M) + 2\ Ag(s)$

$E^0_{cell} \quad 2.97\,V$

E_{cell} for the above reaction is __________ V. (Nearest integer)

[Given : log 2.5 = 0.3979, T = 298 K]

Ans. 3

Sol. $E_{cell} \quad E^0_{cell} \quad \frac{0.059}{2}\log\frac{[Cu^{2}\ ]}{[Ag\ ]^2} \quad 2.97 \quad \frac{0.059}{2}\log\ \frac{0.250}{(10\ ^{3})^2}$

$= 2.97 - 0.177\ (-0.602) = 3.07$

Ans. = 3

4. If the standard molar enthalpy change for combustion of graphite powder is -2.48×10^2 kJ mol^{-1}, the amount of heat generated on combustion of 1 g of graphite powder is _________ kJ. (Nearest integer)

Ans. 21

Sol. $C(graphite) + O_2 \rightarrow CO_2(g)\ \Delta H = -2.48$ kJ mole

1 gram

Total heat released $2.48 \quad \frac{1}{12} \quad 10^2$

= 20.67

Ans. 21

5. The total number of unpaired electrons present in $[Co(NH_3)_6]Cl_2$ and $[Co(NH_3)_6]Cl_3$ is _________.

Ans. 1

Sol. Complex

(i) $[Co(NH_3)_6]\ Cl_2 \Rightarrow Co^{2+} = 3d^7$ unpaired electron = 1

(ii) $[Co(NH3)_6]\ Cl_3 \Rightarrow Co^{3+} = 3d^6$ unpaired electron = 0

Total unpaired electrons = 1

6. If the concentration of glucose ($C_6H_{12}O_6$) in blood is 0.72 g L^{-1}, the molarity of glucose in blood is _________ $\times 10^{-3}$ M. (Nearest integer)

(Given : Atomic mas of C = 12, H = 1, O = 16 u)

Ans. 4

Sol. $M \quad \frac{W_{solute}}{M_{solute} \quad V_{soln}(inlit.)} \quad \frac{0.72}{180}$

$= 0.004 = 4 \times 10^{-3}$

7. The number of acyclic structural isomers (including geometrical isomers) for pentene are __________.

Ans. 6

Sol. C–C–C–C=C (1)

C–C–C=C–C (2)

```
        |
C – C – C = C   (1)

C – C – C = C   (1)
    |

    |
C – C – C = C   (1)
```

8. A copper complex crystallising in a CCP lattice with a cell edge of 0.4518 nm has been revealed by employing X-ray diffraction studies. The density of a copper complex is found to be 7.62 g cm^{-3}. The molar mass of copper complex is _________ g mol^{-1}. (Nearest integer)

[Given : $N_A = 6.022 \times 10^{23}$ mol^{-1}]

Ans. 106

Sol. $d \quad \frac{Z \quad M}{N_A \quad Volume}$

$7.62 \quad \frac{4 \quad M}{6.022 \quad 10^{23} \quad [0.4518 \quad 10^{\ 7}]^3}$

$$M \quad \frac{7.62 \quad 6.022 \quad 10^{23} \quad [0.4518 \quad 10^{7}]^{3}}{4}$$

$= 1.057 \times p10^{2}$

= 105.7 gram/mole

Ans. 106

9. $N_2O_2(g) \quad 2NO_2(g) \quad \frac{1}{2}O_2(g)$

In the above first order reaction the initial concentration of N_2O_5 is 2.40×10^{-2} mol L^{-1} at 318 K. the concentration of N_2O_5 after 1 hour was 1.60×10^{-2} mol L^{-1}. The rate constant of the reaction at 318 K is _________ $\times 10^{-3}$ min^{-1}. (Nearest integer)

[Given : log 3 = 0.477, log 5 = 0.699]

Ans. 7

Sol.

	$2N_2O_5(g)$	$\rightarrow$	$2NO_2(g)$	+	$\frac{1}{2}O_2(g)$
Initial	$2{\cdot}4 \times 10^{-2}$ M		–		–
After 1 hour	$1{\cdot}6 \times 10^{-2}$ M				

$$K \quad \frac{1}{t}\ln \frac{a}{a \quad x}$$

$$K \quad \frac{2.303}{60}\log \frac{2.4 \quad 10^{2}}{1.6 \quad 10^{2}}$$

$$K \quad \frac{2.303}{60}\log \frac{3}{2}$$

$k = 0{\cdot}0069 = 6.9 \times 10^{-3}$ min^{-1}

k = 7

10. Number of electrons that Vanadium (Z = 23) has in p-orbitals is equal to _________.

Ans. 12

Sol. $_{23}V = 1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^3\ 4s^2$

PART C : MATHEMATICS

Single Choice Type

This section contains **20 Single choice questions**. Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The values of λ and μ such that the system of equations

$x + y + z = 6,\ 3x + 5y + 5z = 26,\ x + 2y + \lambda z = \mu$ has no solution, are:

(1) $\lambda = 3, \mu \neq 10$ (2) $\lambda \neq 2, \mu = 10$ (3) $\lambda = 2, \mu \neq 10$ (4) $\lambda = 3, \mu = 5$

Ans. 3

Sol. For no solution $\Delta = 0$

$\Delta = 0$

$$\begin{vmatrix} 1 & 1 & 1 \\ 3 & 5 & 5 \\ 1 & 2 & \end{vmatrix} \quad 0$$

$\Rightarrow 1(5\lambda - 10) - 1(3\lambda - 5) + 1(6 - 5) = 0$

$\Rightarrow 2\lambda - 4 = 0$

$\Rightarrow \lambda = 2$

$$_1 \begin{vmatrix} 6 & 1 & 1 \\ 26 & 5 & 5 \\ & 2 & 2 \end{vmatrix} \quad 0$$

$$_2 \begin{vmatrix} 1 & 6 & 1 \\ 3 & 26 & 5 \\ 1 & & 2 \end{vmatrix} \quad 1(52 \quad 5\ ) \quad 6(6 \quad 5) \quad 1(3 \quad 26)$$

$= 52 - 5\mu - 6 + 3\mu - 26$

$\Delta_2 = 20 - 2\mu$

$$_3 \begin{vmatrix} 1 & 1 & 6 \\ 3 & 5 & 26 \\ 1 & 2 & \end{vmatrix} \quad 1(5 \quad 52) \quad 1(3 \quad 26) \quad 6(6 \quad 5)$$

$\Delta_3 = 2\mu - 20$

Case-I

$\lambda = 2, \mu = 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 = 0, \Delta_3 = 0$

system of equations are

$x + y + z = 6$

$3x + 5y + 5z = 26$

$x + 2y + 2z = 10$ has infinite many solutions

Case – II

$\lambda = 2, \mu \neq 10 \Rightarrow \Delta = 0, \Delta_1 = 0, \Delta_2 \neq 0, \Delta_3 \neq 0$

system has no solution

2. Let $E_1 : \frac{x^2}{a^2} \quad \frac{y^2}{b^2} \quad 1$, a > b. Let E_2 be another ellipse such that it touches the end points of major axis of E_1 and the foci of E_2 are the end points of minor axis of E_1. If E_1 and E_2 have same eccentricities, then its value is:

(1) $\frac{1 \quad \sqrt{6}}{2}$ (2) $\frac{1 \quad \sqrt{3}}{2}$ (3) $\frac{1 \quad \sqrt{5}}{2}$ (4) $\frac{1 \quad \sqrt{8}}{2}$

Ans. 3

Sol. Eccentricity of E_1 is $e \Rightarrow e^2 \quad 1 \quad \frac{b^2}{a^2}$

Eccentricity of E_2 is $e \Rightarrow e^2 \quad 1 \quad \frac{a^2}{k^2}$

So, $e^2 \quad 1 \quad \frac{b^2}{a^2} \quad 1 \quad \frac{a^2}{k^2} \quad k \quad \frac{a^2}{b}$ ….(i)

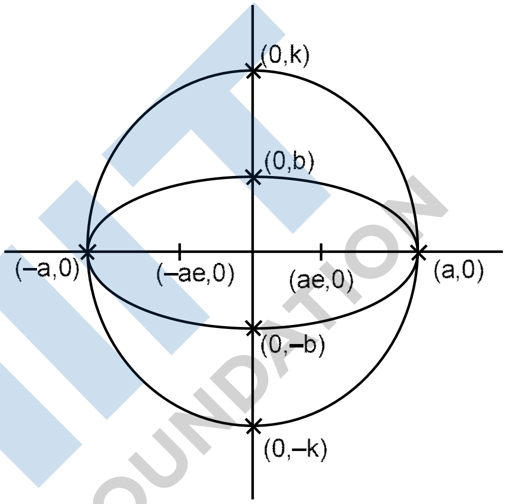
Also ke = b …..(ii)

From equation (i) and (ii) $e \quad \frac{b^2}{a^2}$

Since $e^2 \quad 1 \quad \frac{b^2}{a^2} \quad e^2 \quad 1 \quad e$

$\Rightarrow e^2 + e - 1 = 0$

$e \quad \frac{\sqrt{5} \quad 1}{2}$



3. Let Sn denote the sum of first n- terms of an arithmetic progression. If $S_{10} = 530$, $S_5 = 140$, then $S_{20} - S_6$ is equal to:

(1) 1872 (2) 1842 (3) 1862 (4) 1852

Ans. 3

Sol. $S_{10} = 530$

$\frac{10}{2}[2a \quad 9d] \quad 530$

$\Rightarrow 2a + 9d = 106$ … (1)

$S_5 = 140$

$\frac{5}{2}[2a \quad 4d] \quad 140$

$\Rightarrow 2a + 4d = 56$ … (2)

$\Rightarrow 5d = 50$

d = 10

a = 8

Now,

$\Rightarrow S_{20} - S_6 =$

$\Rightarrow 10[2a + 19d] - 3[2a + 5d]$

$\Rightarrow 14a + 175d$

$14 \times 8 + (175)10 = 1862$

4. Let $f : R \to R$ be defined as $f(x) = \begin{cases} \dfrac{x^3}{(1-\cos 2x)^2}\log_e\left(\dfrac{1+2xe^{-2x}}{\left(1-xe^{-x}\right)^2}\right) & ; x \neq 0 \\ \alpha & ; x = 0 \end{cases}$

If f is continuous at x = 0, then α is equal to:

(1) 2 (2) 3 (3) 1 (4) 0

Ans. 3

Sol. $\lim\limits_{x\to 0}\dfrac{x^3}{4\sin^4 x}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$

$\lim\limits_{x\to 0}\dfrac{1}{4\left(\dfrac{\sin x}{x}\right)^4 x}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)$

$\lim\limits_{x\to 0}\dfrac{1}{4x}\log_e\left(\dfrac{1+2xe^{-2x}}{(1-xe^{-x})^2}\right)\times\dfrac{x^4}{\sin^4 x}$

$\lim\limits_{x\to 0}\dfrac{1}{4}\left[\dfrac{2e^{-2x}\log(1+2xe^{-2x})}{2xe^{-2x}}+\dfrac{2e^{-x}\log(1-xe^{-x})}{-xe^{-x}}\right]\times\dfrac{x^4}{\sin^4 x}$

$\lim\limits_{x\to 0}\dfrac{1}{4}\left[2e^{-2x}+2e^{-x}\right]\times\dfrac{x^4}{\sin^4 x} = \dfrac{2+2}{4} = 1$

If f is continuous at x = 0 then

$f(0) = \lim\limits_{x\to 0} f(x)$

$\alpha = 1$

5. If the shortest distance between the straight lines $3(x-1) = 6(y-2) = 2(z-1)$ and $4(x-2) = 2(y-\lambda) = (z-3)$, $\lambda \in R$ is $\dfrac{1}{\sqrt{38}}$, then the integral value of λ is equal to :

(1) 2 (2) 5 (3) 3 (4) –1

Ans. 3

Sol. Lines are $\dfrac{x-1}{\frac{1}{3}} = \dfrac{y-2}{\frac{1}{6}} = \dfrac{z-1}{\frac{1}{2}}$ and $\dfrac{(x-2)}{\frac{1}{4}} = \dfrac{(y-\lambda)}{\frac{1}{2}} = \dfrac{(z-3)}{1}$

Shortest distance $= \dfrac{1}{\sqrt{38}}$

$\vec{a}_2 \quad 2\hat{i} \quad \hat{j} \quad 3\hat{k}, \vec{a}_1 \quad \hat{i} \quad 2\hat{j} \quad \hat{k}$

$\vec{b}_1 \quad \frac{1}{3}\hat{i} \quad \frac{1}{6}\hat{j} \quad \frac{1}{2}\hat{k}, \vec{b}_2 \quad \frac{1}{4}\hat{i} \quad \frac{1}{2}\hat{j} \quad \hat{k}$

$\vec{b}_1 \quad \vec{b}_2 \quad \frac{1}{12}\hat{i} \quad \frac{5}{24}\hat{j} \quad \frac{1}{8}\hat{k}$

$|\vec{b}_1 \quad \vec{b}_2| \quad \frac{\sqrt{38}}{24}$

$\frac{1}{\sqrt{38}} \quad \left|\frac{(\vec{a}_2 \quad \vec{a}_1)\ (\vec{b}_1 \quad \vec{b}_2)}{|\vec{b}_1 \quad \vec{b}_2|}\right|$

$\frac{1}{\sqrt{38}} \quad \left|\frac{(\hat{i} \quad (\quad 2)\hat{j} \quad 2\hat{k}) \quad \frac{1}{12}\hat{i} \quad \frac{5}{24}\hat{j} \quad \frac{1}{8}\hat{k}}{\frac{\sqrt{38}}{24}}\right| k$

$\frac{1}{24} \quad \left|\frac{1}{12} \quad (\quad 2)\frac{5}{24} \quad \frac{1}{4}\right|$

$\Rightarrow 1 = |14 - 5\lambda|$

$\frac{13}{5}$ (rejected)

$\Rightarrow \lambda = 3$

6. Let $f : R \to R$ be defined as

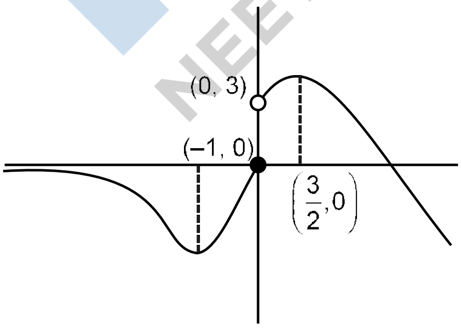
$$f(x) \quad \begin{matrix} \frac{4}{3}x^3 \quad 2x^2 \quad 3x & , \quad x \quad 0 \\ 3xe^x & , \quad x \quad 0 \end{matrix}$$

Then f is increasing function in the interval.

(1) $\frac{1}{2},2$ (2) (–3, –1) (3) (0,2) (4) $1,\frac{3}{2}$

Ans. 4

Sol. $f(x) \quad \begin{matrix} 4x^2 \quad 4x \quad 3 & , \quad x \quad 0 \\ 3(x \ e^x \quad e^x) & , \quad x \quad 0 \end{matrix}$



7. Let a line L : $2x + y = k$, $k > 0$ be a tangent to the hyperbola $x^2 - y^2 = 3$. If L is also a tangent to the parabola $y^2 = \alpha x$, then α is equal to:

(1) 24 (2) –24 (3) 12 (4) –12

Ans. 2

Sol. Given slope of line (m) = – 2

slope form of tangent to the curve $x^2 - y^2 = 3$ is $y \quad mx \quad \sqrt{a^2m^2 \quad b^2}$

$\Rightarrow y = -2x \pm 3$

On comparing, with the equation $2x + y = k$, $(k > 0) \Rightarrow k = 3$

Now, slope form of tangent to the parabola y2 = αx is $y \quad mx \quad \frac{}{4m}$

But m = – 2 so

$y \quad 2x \quad \frac{}{4(\ 2)}$

$3 \quad \frac{}{4 \ (\ 2)}$

$\alpha = -24$

8. Four dice are thrown simultaneously and the numbers shown on these dice are recorded in 2 × 2 matrices. The probability that such formed matrices have all different entries and are non-singular, is:

(1) $\frac{22}{81}$ (2) $\frac{23}{81}$ (3) $\frac{45}{162}$ (4) $\frac{43}{162}$

Ans. 4

Sol. Number of matrices having distinct elements = ${}^6C_4 \times 4!$

$\Rightarrow$ Number of non singular matrices having distinct elements

$= {}^6C_4 \times 4!$ – Number of singular matrices having distinct elements

$X \quad \begin{matrix} a & b \\ c & d \end{matrix}$

$|x| = ad - bc = 0$

$\begin{matrix} (1, 6) & (3, 2) \\ (3, 4) & (6\ 2) \end{matrix} \quad 8 \quad 8$ possibilities

$\Rightarrow$ Number of non singular matrices having distinct elements

$= {}^6C_4 \times 4! - 16 = 344$

So required probability $\frac{344}{6^4} \quad \frac{13}{162}$

9. Let three vectors $\vec{a}, \vec{b}$ and $\vec{c}$ be such that $\vec{a} \quad \vec{b} \quad \vec{c}, \vec{b} \quad \vec{c} \quad \vec{a}$ and $|\vec{a}| \quad 2$. Then which one of the following is not true?

(1) Projection of a on $\vec{a}$ on $\vec{b} \quad \vec{c}$ is 2

(2) $|3\vec{a} \quad \vec{b} \quad 2\vec{c}|^2 \quad 51$

(3) $|\vec{a}\,\vec{b}\,\vec{c}| \quad |\vec{c}\,\vec{a}\,\vec{b}| \quad 8$

(4) $\vec{a} \quad \vec{b} \quad \vec{c} \quad \vec{b} \quad \vec{c} \quad 0$

Ans. 2

Sol. $|\vec{a}| \quad 2$

$\vec{a} \quad \vec{b} \quad \vec{c} \quad [\vec{a}\,\vec{b}\,\vec{c}] \quad |\vec{c}|^2$

$\vec{b} \quad \vec{c} \quad \vec{a} \quad [\vec{a}\,\vec{b}\,\vec{c}] \quad |\vec{a}|^2$

Hence $|\vec{a}|^2 \quad |\vec{c}|^2 \quad 4$

Also $\vec{a} \quad \vec{b} \quad \vec{b} \quad \vec{c} \quad \vec{a} \quad \vec{c} \quad 0$

1. Projection of $\vec{a}$ on $\vec{b} \quad \vec{c}$ $\quad \dfrac{\vec{a} \quad \vec{b} \quad \vec{c}}{|\vec{b} \quad \vec{c}|} \quad \dfrac{\vec{a} \quad \vec{b} \quad \vec{c}}{|\vec{a}|} \quad \dfrac{\vec{a} \quad \vec{b} \quad \vec{c}}{|\vec{a}|} \quad \dfrac{|\vec{c}|^2}{|\vec{a}|} \quad |\vec{a}| \quad 2$ (correct)

2. $|3\vec{a} \quad \vec{b} \quad 2\vec{c}|^2 \quad 9|\vec{a}|^2 \quad |\vec{b}|^2 \quad 4|\vec{c}|^2 \quad 6(\vec{a} \quad \vec{b}) \quad 12(\vec{a} \quad \vec{c}) \quad 4(\vec{b} \quad \vec{c})$

$36 \quad |\vec{b}|^2 \quad 16 \quad 0 \quad 0 \quad 0$

$52 \quad |\vec{b}|^2 \quad 51$ (Incorrect)

3. $\vec{a}\,\vec{b}\,\vec{c} \quad \vec{c}\,\vec{a}\,\vec{b} \quad |\vec{a}|^2 \quad |\vec{a}|^2 \quad 2|\vec{a}|^2 \quad 8$

4. $\vec{a} \quad \vec{b} \quad \vec{c} \quad \vec{b} \quad \vec{c} \quad \vec{a} \quad 2\,\vec{b} \quad \vec{c} \quad 2\vec{a} \quad \vec{a} \quad 0$ (correct)

10. Which of the following Boolean expressions is not a tautology?

(1) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

(2) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

(3) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

(4) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

Ans. 1

Sol. (1) $(\sim p \Rightarrow q) \vee (\sim q \Rightarrow p)$

$\Rightarrow (p \vee q) \vee (q \vee p)$

$\Rightarrow p \vee q$

(2) $(p \Rightarrow \sim q) \vee (\sim q \Rightarrow p)$

$\Rightarrow (\sim p \vee \sim q) \vee (q \vee p)$

$\Rightarrow t$

(3) $(q \Rightarrow p) \vee (\sim q \Rightarrow p)$

$\Rightarrow (\sim q \vee p) \vee (q \vee p)$

$\Rightarrow t$

(4) $(p \Rightarrow q) \vee (\sim q \Rightarrow p)$

$\Rightarrow (\sim p \vee q) \vee (q \vee p)$

$\Rightarrow t$

11. Let n denote the number of solutions of the equation $z^2 \quad 3\bar{z} \quad 0$ where z is a complex number. Then value of $\displaystyle\sum_{k\ 0} \frac{1}{n^k}$ is equal to :

(1) $\frac{3}{2}$ (2) 2 (3) 1 (4) $\frac{4}{3}$

Ans. 4

Sol. Let z = x + iy

$(x + iy)^2 + 3(x - iy) = 0$

$x^2 - y^2 + 2ixy + 3x - 3iy = 0$

$x^2 - y^2 + 3x = 0$ & $2xy - 3y = 0$

Case-1: y = 0

$x^2 - y^2 + 3x = 0$

$\Rightarrow x = 0$ or $x = -3$

Solutions are z = 0 and z = –3

Case-2: $x \quad \frac{3}{2}$

$x^2 - y^2 + 3x = 0$

$y \quad \frac{3\sqrt{3}}{2}$ or $y \quad \frac{3\sqrt{3}}{2}$

Solutions are $z \quad \frac{3}{2} \quad i\frac{3\sqrt{3}}{2}$ and $z \quad \frac{3}{2} \quad i\frac{3\sqrt{3}}{2}$

Total number of solutions = n = 4

So $\displaystyle\sum_{k\ 0} \frac{1}{4^k} \quad \frac{1}{1 \quad \frac{1}{4}} \quad \frac{4}{3}$

12. Let A = $[a_{ij}]$ be a real matrix of order 3 × 3, such that $a_{i1} + a_{i2} + a_{i3} = 1$, for 1, 2, 3. Then, the sum of all the entries of the matrix A^3 is equal to:

(1) 9 (2) 1 (3) 2 (4) 3

Ans. 4

Sol. $A \quad \begin{matrix} a & b & c \\ d & e & f \\ g & h & i \end{matrix}$

a + b + c = 1

d + e + f = 1

g + h + i = 1

Let suppose a matrix $Y = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

So,

$$AY = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} a+b+c \\ d+e+f \\ g+h+i \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

AY = Y ……..(1)

Substitute Y = AY in equation (1)

So, $A^2Y = AY = Y$

Again substitute Y = AY

$\Rightarrow A^3Y = A^2Y = AY = Y$

So, $A^3Y = Y$

Let us suppose $A^3 = \begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}$

$$\begin{pmatrix} A & B & C \\ D & E & F \\ G & H & I \end{pmatrix}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} A+B+C \\ D+E+F \\ G+H+I \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

A + B + C = 1

D + E + F = 1

G + H + I = 1

So, A + B + C + D + E + F + G + H + I = 3

Sum of elements of $A^3 = 3$

13. Let L be the line of intersection of planes $\vec{r}\cdot(\hat{i}-\hat{j}+2\hat{k}) = 2$ and $\vec{r}\cdot(2\hat{i}+\hat{j}-\hat{k}) = 2$. If P (α, β, γ) is the foot of perpendicular on L from the point (1, 2, 0) then the value of $35(\alpha + \beta + \gamma)$ is equal to:

(1) 119 (2) 134 (3) 101 (4) 143

Ans. 1

Sol. Given planes are

x – y + 2z = 2 and 2x + y – z = 2

z = 0

$\Rightarrow$ x – y = 2 and 2x + y = 2

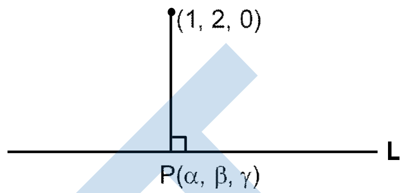
(1) and (2), 3x = 4 $\quad x \quad \frac{4}{3} \quad y \quad \frac{2}{3}$

$\frac{4}{3},\frac{2}{3},0$ lies on line of intersection of planes

for dr's of line $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 2 \\ 2 & 1 & 1 \end{vmatrix}$

$\hat{i}(1 \quad 2) \quad \hat{j}(\quad 1 \quad 4) \quad \hat{k}(1 \quad 2)$

$\hat{i} \quad 5\hat{j} \quad 3\hat{k}$



$\therefore$ line of intersection is

$\frac{x \quad \frac{4}{3}}{1} \quad \frac{y \quad \frac{2}{3}}{5} \quad \frac{z \quad 0}{3}$

$x \quad \frac{4}{3}$ and z = 3λ

$y \quad 5 \quad \frac{2}{3}$

$\frac{4}{3} \quad 1 \quad 1 \quad 5 \quad \frac{2}{3} \quad 2 \quad 5 \quad 3 \quad 3 \quad 0$

$\frac{1}{3} \quad 1 \quad 5 \quad \frac{8}{3} \quad 5 \quad 9 \quad 0$

$\frac{1}{3} \quad 25 \quad \frac{40}{3} \quad 9 \quad 0$

$35 \quad \frac{41}{3} \quad \frac{41}{105}$

so, $\frac{4}{3} \quad \frac{41}{105} \quad \frac{99}{105}$

$5 \quad \frac{41}{105} \quad \frac{2}{3} \quad \frac{205 \quad 70}{105} \quad \frac{135}{105}$

$\frac{123}{105}$

$35(\qquad) \quad \frac{(99 \quad 135 \quad 123)}{105} \quad 35 \quad 119$

14. If $\int_0^{100} \frac{\sin^2 x}{e^{\frac{x}{} \quad \frac{x}{}}} dx \quad \frac{3}{1 \quad 4}$, $\alpha \in$ R, where [x] is the greatest less than or equal to x, then the value of α is:

(1) 50(e – 1) (2) 200 (1 – e^{-1}) (3) 150 (e^{-1} – 1) (4) 100 (1 – e)

Ans. 2

Sol. $\int_0^{100} \frac{\sin^2 x}{e^{\frac{x}{}}} dx$

$100 \int_0 \frac{\sin^2 x}{e^{x/}} dx$

$50 \int_0 e^{x/} (1 \quad \cos 2x)\, dx$

$50\ e^{x/} \quad (\quad)_0 \quad 50 \int_0 e^{x/} \cos 2x\, dx$

$50 \quad (\quad)(e^{1} \quad 1) \quad \frac{50 \quad e^{x/} \left(\frac{1}{} \cos 2x \quad 2\sin 2x \right)_0}{\frac{1}{2} \quad 4}$

$50\ (e^{1} \quad 1) \quad \frac{50\ ^2}{1 \quad 4\ ^2}\ e^{1} \quad \frac{1}{} \quad \frac{1}{}$

$\frac{200\ ^3(1 \quad e^{1})}{1 \quad 4\ ^2}$ So $\alpha = 200(1 - e^{-1})$

15. The number of solution of $\sin^7 x + \cos^7 x = 1$, $x \in [0, 4\pi]$ is equal to :

(1) 11 (2) 9 (3) 5 (4) 7

Ans. 3

Sol. $\sin^2 x + \cos^2 x = 1$, $\sin^2 x \le 1$ and $\cos^2 x \le 1$

$\sin^7 x \le \sin^2 x$

$\cos^7 x \le \cos^2 x$

so, $\sin^7 x + \cos^7 x \le 1$

$\sin^7 x + \cos^7 x = 1$ when $\sin^7 x = \sin^2 x$ & $\cos^7 x = \cos^2 x$

Case-1 : sin x = 0, cos x = 1 $\Rightarrow$ x = 0 , 2π, 4π

Case-2 : sin x = 1, cos x = 0 $\Rightarrow$ x $\frac{}{2}, \frac{5}{2}$

Total number of solutions = 5

16. Let the circle S: $36x^2 + 36y^2 - 108x + 120y + C = 0$ be such that it neither intersects nor touches the co-ordinate axes. If the point of intersection of the lines, x – 2y = 4 and 2x – y = 5 lies inside the circle S, then:

(1) $\frac{25}{9} \quad C \quad \frac{13}{3}$ (2) 100 < C < 165 (3) 100 < C < 156 (4) 81 < C < 156

Ans. 3

Sol. Intersection point of $2x - y = 5$ and $x - 2y = 4$ is $(2, -1)$

So, $(2, -1)$ lies indie the circle $\Rightarrow S_1 < 0$

$36(2)^2 + 36(-1)^2 - 108(2) + 120(-1) + c < 0$

$c < 156$ ....(i)

$\because$ circle $36x^2 + 36y^2 - 108x + 120y + c = 0$ neither touches nor cuts the co-ordinate axis so

$g^2 \quad c \quad 0 \quad \frac{3}{2}^{\,2} \quad \frac{c}{36} \quad 0 \quad c \quad 81$ ....(ii)

and $f^2 \quad c \quad 0 \quad \frac{5}{3}^{\,2} \quad \frac{c}{36} \quad 0 \quad c \quad 100$ ....(iii)

From (i), (ii) and (iii)

$100 < c < 156$

17. Let a vector $\vec{a}$ be coplanar with vectors $\vec{b} \quad 2\hat{i} \quad \hat{j} \quad \hat{k}$ and $\vec{c} \quad \hat{i} \quad \hat{j} \quad \hat{k}$. If $\vec{a}$ is perpendicular to $\vec{d} \quad 3\hat{i} \quad 2\hat{j} \quad 6\hat{k}$, and $|\vec{a}| \quad \sqrt{10}$. Then a possible value of $\vec{a}\,\vec{b}\,\vec{c} \quad \vec{a}\,\vec{b}\,\vec{d} \quad \vec{a}\,\vec{c}\,\vec{d}$ is equal to :

(1) – 42 (2) – 38 (3) – 40 (4) – 29

Ans. 1

Sol. Let $\vec{a} \quad \vec{b} \quad \vec{c}$. Where $\vec{a}\,\vec{b}\,\vec{c}$ are coplanar.

Now $\vec{a} \quad \vec{b} \quad \vec{a} \quad \vec{d} \quad 0$

$\vec{b} \quad \vec{d} \quad (\vec{c} \quad \vec{d}) \quad 0$

$\Rightarrow (6 + 2 + 6) + \lambda(3 - 2 + 6) = 0$

$\Rightarrow 14 + 7\lambda = 0$

$\Rightarrow \lambda = -2$

$\vec{a} \quad \vec{b} \quad 2\vec{c} \quad 3\hat{j} \quad \hat{k}$

Now $\vec{a}\,\vec{b}\,\vec{c} \quad \vec{a}\,\vec{b}\,\vec{d} \quad \vec{a}\,\vec{c}\,\vec{d}$

$0 \quad (\vec{a} \quad \vec{d}) \quad \vec{b} \quad (\vec{a} \quad \vec{d}) \quad \vec{c}$

$(\vec{a} \quad \vec{d}) \quad (\vec{b} \quad \vec{c})$

$(\vec{a} \quad \vec{d}) \quad (3\hat{i} \quad 2\hat{k})$

$$\begin{vmatrix} 0 & 3 & 1 \\ 3 & 2 & 6 \\ 3 & 0 & 2 \end{vmatrix}$$

$= -42$

18. Let $[x]$ denote the greatest integer less than or equal to x. Then, the value of $x \in R$ satisfying the equation $[e^x]^2 + [e^x + 1] - 3 = 0$ lie in the interval :

(1) [0,1/e] (2) [1,e) (3) $[\log_e 2, \log_e 3)$ (4) $[0, \log_e 2)$

Ans. 4

Sol. $[e^x]^2 + [e^x+1] - 3 = 0$

$[e^x]^2 + [e^x] - 2 = 0$

Let $[e^x] = t$

$t^2 + t - 2 = 0$

$(t + 2)(t - 1) = 0$

$t = 1, -2$

$[e^x] : 1, -2$ (–2 is not possible)

$[e^x] = 1$

$x \in [0, \ln 2)$

19. Let y = y(x) be the solution of the differential equation $\operatorname{cosec}^2 x\, dy + 2dx = (1 + y\cos 2x)\operatorname{cosec}^2 x\, dx$, with

$y \; \frac{}{4} \quad 0$, then the value of $(y(0) + 1)^2$ is equal to :

(1) $e^{1/2}$ (2) e^{-1} (3) e (4) $e^{-1/2}$

Ans. 2

Sol. $\operatorname{cosec}^2 x dy + 2dx = (1+y\cos 2x)\operatorname{cosec}^2 x\, dx$

$$\frac{dy}{dx} \quad 2\sin^2 x \quad 1 \quad y\cos 2x$$

$$\frac{dy}{dx} \quad y\cos 2x \quad 1 \quad 2\sin^2 x$$

$$\frac{dy}{dx} \quad \cos 2x(1 \quad y)$$

$$\frac{dy}{(1 \quad y)} \quad \cos 2x\, dx$$

$$\log(1 \quad y) \quad \frac{\sin 2x}{2} \quad c$$

Given $y \; \frac{}{4} \quad 0$

$$\log \; 1 \quad y \; \frac{}{4} \quad \frac{\sin \frac{}{2}}{2} \quad c$$

$$c \quad \frac{1}{2}$$

$$\text{Now } \log \; 1 \quad y(0) \quad \frac{\sin 0}{2} \quad \frac{1}{2}$$

$$1 \quad y(0) \quad e^{\frac{1}{2}}$$

$(1+y(0))^2 = e^{-1}$

20. If the domain of the function $f(x) = \dfrac{\cos^{-1}\sqrt{x^2 - x + 1}}{\sqrt{\sin^{-1}\left(\dfrac{2x-1}{2}\right)}}$ is the interval $(\alpha, \beta]$, then $\alpha + \beta$ is equal to :

(1) 2 (2) $\frac{3}{2}$ (3) $\frac{1}{2}$ (4)1

Ans. 2

Sol. $0 \le x^2 - x + 1 \le 1$ and $0 < \dfrac{2x-1}{2} \le 1$

$\Rightarrow$ $x(x-1) \le 0$ & $1 < 2x \le 3$

$\Rightarrow$ $x \in [0,1] \cap x \in \left(\dfrac{1}{2}, \dfrac{3}{2}\right]$

$\Rightarrow$ $x \in \left(\dfrac{1}{2}, 1\right]$

Hence $\alpha + \beta = \dfrac{1}{2} + 1 = \dfrac{3}{2}$

Numeric Value Type

This Section contains **10 Numeric Value Type question**, out of 10 only 5 have to be done.

1. Let A = {0,1,2,3,4,5,6,7} Then the number of bijective functions $f : A \to A$ such that $f(1) + f(2) = 3 - f(3)$ is equal to :

Ans. 720

Sol. $f(1) + f(2) = 3 - f(3)$

$\Rightarrow f(1) + f(2) + f(3) = 3$

$\Rightarrow \{(f(1),f(2),f(3)\} = \{(0,1,2)\ (0,2,1)\ (1,0,2)(1,2,0)(2,1,0)(2,0,1)\} = 3! = 6$

and $\{f(0), f(4), f(5), f(6), f(7)\} = 5!$

Total such function = $5! \times 3! = 720$

2. Consider the following frequency distribution :

Class :	0-6	6-12	12-18	18-24	24-30
Frequency :	a	b	12	9	5

If mean $\frac{309}{22}$ and median = 14, then the value $(a - b)^2$ is equal to.

Ans. 4

Sol.

Midpoint	Frequency	Cumulative freq.
3	a	a
9	b	a + b
15	12	a + b + 12
21	9	a + b + 21
27	5	a + b + 26
		n = a + b + 26

$\text{mean} \quad \frac{3a \quad 9b \quad 180 \quad 189 \quad 135}{a \quad b \quad 26} \quad \frac{309}{22}$

$\Rightarrow 81a + 37b = 1018$ ....(1)

$\text{median} \quad L \quad \frac{\frac{n}{2} \quad cf}{f} \quad h$

$14 \quad 12 \quad \frac{\frac{a \quad b}{2} \quad 13 \quad (a \quad b)}{12} \quad 6$

$\Rightarrow a + b = 18$ .....(2)

Solving (1) & (2), we get a = 8, b = 10

$\Rightarrow (a - b)^2 = 4$

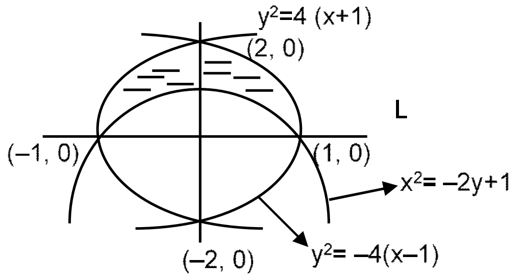
3. The area (in sq. units) of the region bounded by the curves $x^2 + 2y - 1 = 0$, $y^2 + 4x - 4 = 0$ and $y^2 - 4x - 4 = 0$, in the upper half plane is :

Ans. 2

Sol. $x^2 + 2y - 1 = 0$ $\quad$ $y^2 + 4x - 4 = 0$

$y^2 - 4x - 4 = 0$

$y^2 = 4(x+1)$



Area of common region is

$$A \;\; 2 \int_0^1 \sqrt{4 \;\; 4x} \;\; \frac{1 \;\; x^2}{2} \; dx \qquad \text{let } 4 - 4x = t^2$$

$$-4dx = 2tdt$$

$$A \;\; 2 \int_2^0 t \;\; \frac{t}{2} \; dx \;\; \frac{x}{2} \;\; \frac{x^3}{6} \Big|_0^1$$

$$A \;\; 2 \int_0^2 \frac{t^2}{2} dx \;\; 2 \;\; \frac{1}{2} \;\; \frac{1}{6}$$

$$2\frac{2^3}{6} \;\; \frac{4}{6} \;\; \frac{12}{6} \;\; 2$$

4. Let $y = y(x)$ be the solution of the differential equation $x \;\; 2 \; e^{\frac{y \;\; 1}{x \;\; 2}} \;\; y \;\; 1 \; dx \;\; x \;\; 2 \; dy$, $y(1) = 1$. If the domain of $y = y(x)$ is an open interval (α, β), then $|\alpha + \beta|$ is equal to -

Ans. 4

Sol. $x \;\; 2 \; e^{\frac{y \;\; 1}{x \;\; 2}} \;\; y \;\; 1 \; dx \;\; x \;\; 2 \; dy$

$x + 2 = X \qquad \Rightarrow dx = dX$

$y + 1 = Y \qquad \Rightarrow dy = dY$

$$Xe^{\frac{Y}{X}} \;\; Y \;\; dX \;\; XdY$$

$$\frac{dY}{dX} \;\; e^{\frac{Y}{X}} \;\; \frac{Y}{X}$$

$$\text{Put } Y \;\; tX \;\; \frac{dY}{dX} \;\; t \;\; X\frac{dt}{dX}$$

$$t \;\; X\frac{dt}{dX} \;\; e^t \;\; t$$

$$e^t dt \;\; \frac{dX}{X}$$

$\Rightarrow -e^{-t} = \ln|X| + \ln|c|$

$\Rightarrow \ln|cX| = -e^{-t}$

$\Rightarrow \ln(-\ln|cX|) = -t$

$y + 1 = -(x + 2)\ \ln(-\ln|c(x + 2)|)$

$\ln|c(x + 2)| < 0$

$|c(x + 2)| < 1 = -1 < c(x + 2) < 1$

Case -1 $\quad c > 0$

$$\frac{1}{C}\quad x\quad 2\quad \frac{1}{C}$$

$$\frac{1}{C}\quad 2\quad x\quad \frac{1}{C}\quad 2$$

Domain : $\frac{1}{C}\quad 2, \frac{1}{C}\quad 2 \qquad | \qquad | \quad 4$

Case -2 $\quad c < 0$

$$\frac{1}{C}\quad 2\quad x\quad \frac{1}{C}\quad 2$$

Domain : $\frac{1}{C}\quad 2, \frac{1}{C}\quad 2 \qquad | \qquad | \quad 4$

Hence $|\alpha + \beta| = 4$

5. If the constant term, in binomial expansion of $2x^r \quad \frac{1}{x^2} \quad ^{10}$ is 180, then r is equal to -

Ans. 8

Sol. $T_{k+1} = {}^{10}C_k\ (2x^r)^{10-k}\ (x)^{-2k} \Rightarrow {}^{10}C_k\ 2^{(10-k)}.\ x^{10r-rk-2k}$

Now, $\quad 10r - rk - 2k = 0 \Rightarrow r \quad \frac{2k}{10 \quad k}$

And $\quad {}^{10}C_k\ (2)^{10-k} = 180 \Rightarrow k = 8$

$$r \quad \frac{2 \quad 8}{10 \quad 8} \quad 8$$

6. Let $f : R \to R$ be a function defined as $f(x) \quad 3 \quad 1 \quad \frac{|x|}{2} \quad$ if $\ |x| \quad 2$; $\quad 0 \quad$ if $\ |x| \quad 2$

Let $g : R \to R$ be given by $g(x) = f(x + 2) - f(x - 2)$. If n and m denote the number of points in R where g is not continuous and not differentiable, respectively, then n + m is equal to :

Ans. 4

Sol. $f(x) \quad 3 \quad 1 \quad \frac{|x|}{2} \quad$ if $\ |x| \quad 2$

$\qquad 0 \quad$ if $\ |x| \quad 2$

So, $f(x \quad 2) \quad 3 \quad 1 \quad \frac{|x \quad 2|}{2} \quad$ if $\ |x \quad 2| \quad 2$

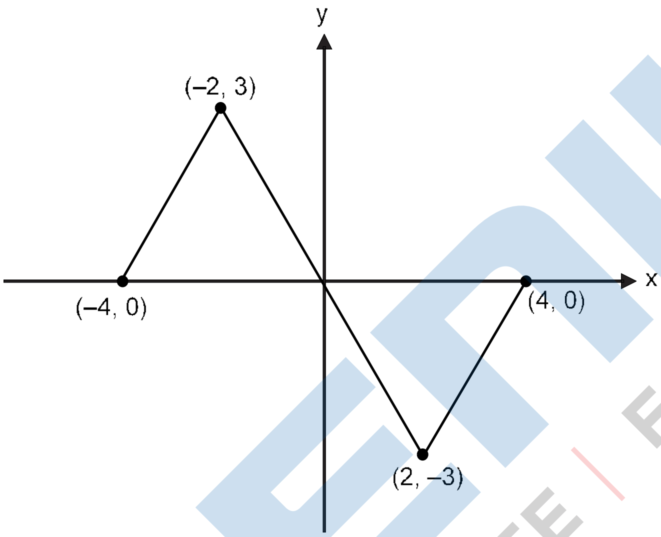
$\qquad 0 \quad$ if $\ |x \quad 2| \quad 2$

$$f(x+2) = \begin{cases} 3\left(1-\dfrac{|x+2|}{2}\right) & \text{if } x \in [-4,0] \\ 0 & \text{if } x \in (-\infty,-4) \cup (0,\infty) \end{cases}$$

$$\text{Similarly } f(x-2) = \begin{cases} 3\left(1-\dfrac{|x-2|}{2}\right) & \text{if } x \in [0,4] \\ 0 & \text{if } x \in (-\infty,0) \cup (4,\infty) \end{cases}$$

g(x) = f(x + 2) – f(x –2)

$$\text{So, } g(x) = \begin{cases} 3\left(1-\dfrac{|x+2|}{2}\right) & ; \quad x \in [-4,0] \\ -3\left(1-\dfrac{|x-2|}{2}\right) & ; \quad x \in [0,4] \\ 0 & ; \quad x \in (-\infty,-4) \cup (4,\infty) \end{cases}$$



number of discontinuous points (n) = 0

Number of non-differentiable point (m) = 4

n + m = 4

7. The number of elements in the set $\{n \in \{1,2,3,\ldots\ldots\ldots,100\} \mid (11)^n > (10)^n + (9)^n\}$ is :

Ans. 96

Sol. Let $11^n > 10^n + 9^n$ $\quad n \in \{1,2,3,\_\_\_\_\ 100\}$

$\Rightarrow 11^n - 9^n > 10^n$

$\Rightarrow (10+1)^n - (10-1)^n > 10^n$

$\Rightarrow 2\,[{}^nC_1\, 10^{n-1} + {}^nC_3\, 10^{n-3} + {}^nC_5\, 10^{n-5} + \ldots\ldots] > 10^n$

$\Rightarrow \frac{1}{5}[{}^nC_1 10^n + {}^nC_3 10^{10-2} + {}^nC_5 10^{10-4} + \ldots\ldots] > 10^n$

$\Rightarrow \frac{1}{5}[{}^nC_1 + {}^nC_3 10^{-2} + {}^nC_5 10^{-4} + \ldots\ldots] > 1$

Clearly the above inequality is true for $n \geq 5$

For n = 4 we have $\frac{1}{5}$ 4 $\frac{4}{10^2}$ $\frac{4}{5}$ $\frac{101}{100}$ 1, Rejected

Hence, number of such n∈{1,2,3,_ _ _ _ 100} is equal to 96

8. The sum of all the elements in the set {n ∈ {1,2,......... 100} | H.C.F. of n and 2040 is 1} is equal to :

Ans. 1251

Sol. 2040 = 23.31. 51. 17

Hence n cannot be multiple of 2,3,5 and 17

Then sum is n(1) – (n(2) + n(3) + n(5) + n(17) – n(6) – n(10) – n(34) – n(15) – n(51) – n(85)) + n(30))

Where n(a) means the sum of all numbers belonging to the set {1,2,......100} which are divisible by a

$\frac{100 \quad 101}{2}$ $\frac{2 \quad 50 \quad 51}{2}$ $\frac{3 \quad 33 \quad 34}{2}$ $\frac{5 \quad 10 \quad 21}{2}$ $\frac{17 \quad 5 \quad 6}{2}$ $\frac{6 \quad 16 \quad 17}{2}$ $\frac{10 \quad 10 \quad 11}{2}$

$\frac{34 \quad 2 \quad 3}{2}$ $\frac{15 \quad 6 \quad 7}{2}$ 51 85 180

= 5050 – 2550 – 1683 – 1050 – 255 + 816 + 550 + 102 + 315 + 51 + 85 – 180

= 1251

9. If the digits are not allowed to repeat in any number formed by using the digits 0,2,4,6,8,then the number of all numbers greater than 10,000 is equal to :

Ans. 96

Sol. Total number = 4 × 4 × 3 × 2 × 1 = 96

10. Let A $\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$. Then the number of 3 × 3 matrices B with entries from the set {1,2,3,4,5} and satisfying AB = BA is :

Ans. 3125

Sol. A $\begin{matrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{matrix}$

Let B $\begin{matrix} a & b & c \\ p & q & r \\ x & y & z \end{matrix}$

Now AB = BA

$\begin{matrix} p & q & r \\ a & b & c \\ x & y & z \end{matrix} \quad \begin{matrix} b & a & c \\ q & p & r \\ y & x & z \end{matrix}$

⇒ p = b , a = q, r = c, x = y, & z = z

Hence number of such matrices are 5^5 = 3125